Oral Exam Questions

Advanced Quantum Field Theory

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1. *What is the input of a path integral?*

   The classical action of the theory.

2. *Write down the Lagrangian of ϕ^4-theory!*

   $\mathcal{L}_{\phi^4} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$.

3. *What is an effective action? Why is it powerful? What are the differences and analogies to the classical action?*

   - Differences: The quantum effective action allows to compute scattering processes that include all quantum corrections at tree level! It is extremely powerful in that all information about a theory's quantum behavior is contained in it.
   - Analogies: The first derivative gives the equation of motion. The second derivative gives the inverse propagator.

4. *What is the difference between $W[J]$ and $\Gamma[\varphi]$?*

   $\Gamma[\varphi]$ is obtained by a Legendre transformation of $W[J]$. $W[J]$ is the generating functional of connected Green's functions, whereas $\Gamma[\varphi]$ generates 1PI diagrams.

5. *Why is the two-point correlation function special?*

   It is the propagator. Its first pole gives the mass of the particle.

6. *Could you write down the partition function for fermionic fields?*

   It is $Z[\boldsymbol{\eta}, \bar{\boldsymbol{\eta}}] = \int \mathcal{D}\bar{\boldsymbol{\psi}}\mathcal{D}\boldsymbol{\psi}\, e^{iS[\boldsymbol{\psi},\bar{\boldsymbol{\psi}}] + i\bar{\boldsymbol{\psi}}\cdot\boldsymbol{\eta} + i\bar{\boldsymbol{\eta}}\cdot\boldsymbol{\psi}}$.

7. *Why do we represent fermions with Grassmann variables?*

   Due to their anticommuting Fermi statistics.

8. *Write down the Coleman-Weinberg potential. What are the steps to derive it?*

   At one-loop, the potential is $V(\varphi)\big|_{\text{1-loop}} = -\frac{i}{2}\int_{\mathbb{R}^{1,3}} \frac{\mathrm{d}k^4}{(2\pi)^4} \ln\left(\frac{\boldsymbol{k}^2 - m^2 - i\epsilon - \frac{\lambda}{2}\varphi_0^2}{\boldsymbol{k}^2 - m^2 - i\epsilon}\right)$. It follows from the background field method which decomposes the field into $\phi(x) = \varphi(x) + f(x)$, i.e. into the sum of its vacuum expectation value $\varphi(x)$ and its quantum fluctuations $f(x)$. Then a loop expansion to first order of $\Gamma[\varphi] = -\mathrm{Vol}_{\mathbb{R}^{1,3}}\, V(\varphi)$ yields the above potential.

9. *Is ϕ^3-theory renormalizable? Why? Is the scalar potential well-defined?*

   Since $[\lambda] = 1$, it is super-renormalizable. Its potential is not well-defined since ϕ^3 is not bounded from below.

10. *What is the relation between the renormalizability of a theory and the dimension of its couplings.*

    The superficial degree of divergence D for a diagram with V vertices is $D \supset [\lambda]\, V$. A theory with coupling λ is therefore called non-renormalizable if $[\lambda] < 0$, renormalizable if $[\lambda] = 0$, and super-renormalizable if $[\lambda] > 0$.

11. *Why do we introduce the so-called counterterms?*

    To absorb divergences appearing in the perturbative calculation of scattering amplitudes.

12. *How many counterterms would we need to introduce in the case of a non-renormalizable, renormalizable, and super-renormalizable theory, if we want to completely renormalize it?*

    - A non-renormalizable theory requires infinitely many counterterms because every diagram is divergent at high enoguh loop-order.
    - A normalizable theory requires finitely many counterterms but these counterterms have to absorb divergences at every loop-order.
    - A super-renormalizable theory not only has finitely many divergent amplitudes but even finitely many divergent Feynman diagrams. That means at some high enough loop order, there are no more divergences to absorb.

13. *What is the β-function? What is a fixed point? Do you know anything about phase-transitions?*

    The β-function describes the running of the coupling, i.e. whether it increases or decreases with growing renormalization scale μ. At a fixed-point of a coupling λ, we have $\beta(\lambda) = 0$.

14. *Give an example of a theory with negative β-function!*

    QCD.

15. *Draw the β-function for QED.*

    At one-loop order, the QED β-function is $\beta(e) = \frac{e^3}{12\pi^2}$.

16. *What is a Landau-Pole?*

    A point at finite renormalization scale μ in the RG flow where the coupling diverges. According to perturbation theory and lattice calculations, this occurs in QED.

17. *Could you tell me something about symmetry breaking? Could we trigger the decay to the true vacuum of a potential by considering quantum corrections?*

    Yes. That is why it is so important to consider quantum corrections and compute the symmetry breaking from the quantum potential as opposed to the classical one.

18. *What does the Higgs potential look like?*

    Like a Mexican hat.

19. *What is an effective field theory? Can you name an example?*

    An effective field theory is valid only under certain assumptions or for certain parameter ranges but breaks down at some point, usually in the high-energy limit, because it does not correctly model the microscopic degrees of freedom. The standard model is an effective field theory.

20. *How do we get rid off the ultraviolet degrees of freedom?*

    By regularization. Most commonly used are cutoffs, dimensional regularization and Pauli-Villars regularization.

21. *Where is the information from the UV degrees of freedom hidden in the effective Lagrangian?*

    In the physical couplings given by the difference of bare couplings and counterterms.

22. *Why are the effective theories so powerful?*

    Effective theories allow us to do physics in the sense that they produce macroscopically precise predictions even though we don't know what is going on at the microscopic level.

23. *Concerning quantization of gauge fields; they are special, aren't they? What can you tell me about that?*

    Gauge theories contain redundancy, i.e. unphysical degrees of freedom. Many field configurations that look different to the theory are completely equivalent to the physical world.

24. *Why does the path integral of a gauge field diverge? What can we do to fix that? (Try to reproduce the case of an Abelian theory , i.e. QED).*

    The naive path integral summing over all field configurations overcounts due to the gauge redundancy. We need to gauge fix the path integral so as to pick only one representative field out of every equivalence class.

25. *What about non-Abelian theories? We cannot pull out the Jacobian determinant from the integral, right? What should we do to get rid of the redundant degrees of freedom?*

    Faddev-Popov ghosts can be used to write the determinant as a Gaussian Berezin path integral. Adding this integration to the path integral over the gauge field gives a new partition function in which the ghosts actually count as negative degrees of freedom that cancel the gauge freedom.

26. *Are ghosts real particles? Why can we not observe them?*

    They are not real. We can't observe them because they violate the spin-statistics theorem (they are bosons with spin 0 but obey Fermi statistics) and thus form states of negative norm. Observing them would violate unitarity.