Quantum Field Theory II
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Assignment 8

Due: Week beginning 08.06.2015.

Problem 8.1 (Superficial degree of divergence for QED):
Review the Feynman rules of quantum electrodynamics and derive the superficial degree of divergence D for this theory. Proceed similarly to the derivation of D for the ϕ^4-theory as it was presented in the lecture.

Problem 8.2 (One-loop structure of QED):
From the question above, you should have found that the superficial degree of divergence of an amplitude, in QED with an interaction term $\sim \overline{\psi}\gamma^\mu A_\mu \psi$, is

$$D = 4 - E_\gamma - \frac{3}{2}E_e, \tag{1}$$

where

$$\begin{aligned} E_\gamma &= \text{number of external photons} \\ E_e &= \text{number of external electrons.} \end{aligned} \tag{2}$$

Since $[e] = 0$, the theory is renormalisable.

a.) Give D for each of the following amplitudes:

i.) The vacuum energy.

ii.) The photon propagator.

iii.) The electron propagator.

iv.) The electron-electron-photon vertex.

v.) The 1-photon amplitude.

vi.) The 3-photon amplitude.

vii.) The 4-photon amplitude.

b.) From general principles one can argue that the photon one-point and three-point functions of quantum electrodynamics vanish, while the four-point function is finite. Verify explicitly that the one-loop diagram contributing to the one-point function vanishes using the discrete symmetry of charge conjugation,

$$j^\mu \to -j^\mu \qquad\qquad A^\mu \to -A^\mu. \tag{3}$$

There are two Feynman diagrams contributing to the three-point function at one-loop order. Show that these cancel. Show that the diagrams contributing to any n-point photon amplitude, for n odd, cancel in pairs.

c.) The photon four-point amplitude is a sum of six diagrams. Show explicitly that the potential logarithmic divergences of these diagrams cancel.

Problem 8.3 (**Reading Assignments**)**:**
This week we're discussing the one-loop structure of QED, thus, it will be useful to read the corresponding chapters in Peskin and Schröder:

a.) Chapter 10 is the relevant chapter for this section of the course. This week, read the first 3 subsections, up to and including 10.3, to clarify your understanding of the current topic.

b.) **Bonus reading material** To answer some questions of previous weeks, the final subsection of the previous chapter, Chapter 9, subsection 9.6, is helpful.