Assignment 9

Due: Week beginning 15.06.2015.

Problem 9.1 (Renormalisation of Yukawa theory):
Consider the pseudoscalar Yukawa Lagrangian,

$$\mathcal{L} = \tfrac{1}{2}\,\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{2}\,m^2\,\phi^2 + \bar{\psi}(i\partial\!\!\!/ - M)\psi - i\,g\,\phi\,\bar{\psi}\,\gamma^5\,\psi\,, \tag{1}$$

where ϕ is a real scalar field and ψ is a Dirac fermion. Notice that this Lagrangian is invariant under the parity transformation $\psi(t,\mathbf{x}) \to \gamma^0\psi(t,-\mathbf{x})$ and $\phi(t,\mathbf{x}) \to -\phi(t,-\mathbf{x})$, in which the field ϕ carries odd parity.

a.) Determine the superficially divergent amplitudes and work out the Feynman rules for renormalised perturbation theory for this Lagrangian. Include all necessary counterterm vertices. Show that the theory contains a superficially divergent $4\,\phi$ amplitude. This means that the theory cannot be renormalised unless one includes a scalar self-interaction,

$$\delta\mathcal{L} = \frac{\lambda}{4!}\phi^4\,, \tag{2}$$

and a counterterm of the same form. It is of course possible to set the renormalised value of this coupling to zero, but that is not a natural choice, since the counterterm will still be non-zero. Are any further interactions required?

b.) Compute the divergent part (the pole as $d \to 4$) of each counterterm, to the one-loop order of perturbation theory, implementing the renormalisation conditions that you specified in part a.). You need not worry about finite parts of the counterterms. Since the divergent parts must have a fixed dependence on the external momenta, you can simplify this calculation by choosing the momenta in the simplest possible way.

Problem 9.2 (Asymptotic behaviour of diagrams in ϕ^4-theory):
Compute the leading terms in the S-matrix element for boson-boson scattering in ϕ^4-theory in the limit $s \to \infty$, t fixed. Ignore all masses on internal lines, and keep external masses non-zero only as infrared regulators where these are needed. Show that

$$i\,\mathcal{M}(s,t) \sim -i\,\lambda - i\frac{\lambda^2}{(4\,\pi)^2}\log s - i\frac{5\,\lambda^3}{2\,(4\,\pi)^4}\log^2 s + \cdots\,. \tag{3}$$

Notice that ignoring the internal masses allows some pleasing simplifications of the Feynman parameter integrals.