

1	2	3	4	$\Sigma$

General Relativity (MKTP3) Summer Term 2015

Exercise sheet 12

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Due: 9:15h, 6 July 2015

1. (15 points) **Lie derivative**

In the lecture, we've seen how the Lie derivative  $(\mathcal{L}_a v)^\mu$  was established via a coordinate transform of  $\bar{v}^\mu(\bar{x})$  to  $v^\mu(x)$  for the r.h.s., and an expansion of  $\bar{v}^\mu$  around  $\bar{x}$  for the l.h.s.

- (a) Derive the Lie derivative of a  $(1, 1)$ -tensor  $Z^\mu_\nu$  in the same fashion,
- (b) and compare it to the one given in the script for the  $(0, 2)$ -tensor  $g_{\mu\nu}$ .  
What's the difference?

2. (15 points) **Killing vectors**

The Lie-derivative of the metric  $g_{\mu\nu}$  is given by

$$\mathcal{L}_a g_{\mu\nu} = g_{\mu\lambda} \nabla_\nu a^\lambda + g_{\lambda\nu} \nabla_\mu a^\lambda = \nabla_\nu a_\mu + \nabla_\mu a_\nu,$$

where  $\nabla_\mu$  are covariant derivatives.

For a Killing vector  $\xi$ , we have

$$\mathcal{L}_\xi g_{\mu\nu} = 0.$$

- (a) For our best friend from this semester, the 2-sphere, where  $ds^2 = d\theta^2 + \sin^2 \theta d\phi^2$ , show that the Killing vector field is related to what we like to call angular momentum.
- (b) Show that for a Killing vector  $\xi$  and the stress-energy-tensor  $T$ , we can define a current  $J^\mu = \xi_\nu T^{\mu\nu}$ , which has a vanishing covariant derivative, i.e.

$$\nabla_\mu J^\mu = 0.$$

3. (10 points) **Real world physics**

- (a) Go outside and enjoy the sun.
- (b) Have some ice cream<sup>1</sup> or something, it's bloody hot outside.
- (c) Do not use this sheet or any upcoming exam as an excuse if anyone asks you to go out this weekend.

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<sup>1</sup>I don't want to suggest alcoholic beverages, that would be reckless

4. (5 points) **Extra: Torsion and cosmology**

- What would happen to Killing vectors if  $\Gamma_{\mu\nu}^{\alpha} \neq \Gamma_{\nu\mu}^{\alpha}$ ?
- Is there a timelike Killing vector in FLRW-cosmologies?

*“To show this diagram properly, I would really need a four dimensional screen. However, because of government cuts, we could manage to provide only a two dimensional screen.”*  
–**Stephen Hawking**